DIFFERENTIAL GRADED DOWN-UP ALGEBRAS AND THEIR ISOMORPHISM PROBLEM

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Abstract

A differential graded (DG for short) down-up algebra is a cochain DG algebra whose underlying graded algebra is a graded down-up algebra. Let $(\mathcal{A}, \partial_{\mathcal{A}})$ be a DG down-up algebra such that its underlying graded algebra $\mathcal{A}^{\#}$ is generated by x, y and subject to the relations

$$x^2 y - \alpha xyx - \beta yx^2 = xy^2 - \alpha yxy - \beta y^2 x = 0,$$

where $\alpha \in \Bbbk$ and $\beta \in \Bbbk^{\times} = \Bbbk \backslash \{0\}$. We give a description of all possible differential of $\mathcal{A}$. In particular, we prove that $\partial_{\mathcal{A}} = 0$ unless $1 + \alpha - \beta = 0$ and $\beta^3 = 1$. Besides the differential structures of DG down-up algebras, we also compute the DG automorphism groups of DG down-up algebras and study the question of when two DG down-up algebras with non-trivial differential are isomorphic.